Hence the decomposition of the given determinant of order six into algebraically irreducible factors is as follows: *

$$D=(\mathrm{I}+\alpha+\beta+\gamma+\delta+\varepsilon)(\mathrm{I}+\alpha+\beta-\gamma-\delta-\varepsilon)\triangle^2.$$

In particular, D is expressible as the difference of two squares,

$$D=[(\mathrm{I}+\alpha+\beta)\triangle]^2-[(\gamma+\delta+\epsilon)\triangle]^2.$$

The fact that $\triangle$ is a factor of D may be shown directly. Multiply the second row of D by ω and the third row by ω^2 and add the products to the first row. Multiply the fifth row by ω^2 and the sixth row by ω and add the products to the fourth row. The new first and fourth rows are

$$\begin{matrix} y & \omega y & \omega^2 y & z & \omega^2 z & \omega z, \\ w & \omega w & \omega^2 w & x & \omega^2 x & \omega x. \end{matrix}$$

Then develop the determinant by Laplace's method. Each term is the product of a determinant of order two formed from the above two rows by the complementary determinant of order four formed from the remaining rows. Each of these determinants of order two has the factor $xy-zw\equiv\triangle$. Hence D has the factor $\triangle$.

The University of Chicago, January, 1902.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

153. Proposed by J. C. CORBIN, Pine Bluff, Ark.

Find some two-figure numbers, such that if they be squared, then the figures interchanged and the resulting numbers squared, the resulting products will consist of the same digits in reversed order.

Solution by G. B. M. ZERR, A.M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

The difference of the two numbers is a multiple of 9.

$\therefore$ $10x+y=10y+x+9n$.

$\therefore$ $x=y+n$.

Let $n=0$. The numbers are 11, 11; 22, 22.

*Since writing this paper, I find that the result was obtained in 1886 by Dedekind by the theory of matrices (Berliner Sitzungsberichte, 1897, page 1007).

Let $n=1$. The numbers are 21, 12.
Let $n=2$. The numbers are 31, 13.
Let $n=1.11$, $y=0$. The numbers are 10, .01.
Let $n=2.22$, $y=0$. The numbers are 20, .02.
Let $n=3.33$, $y=0$. The numbers are 30, .03.

Also solved by *J. H. DRUMMOND* and *J. K. ELLWOOD.*

154. Proposed by J. SCHEFFER, A. M., Hagerstown. Md.

Suppose there is a meadow of 8 acres in which the grass grows uniformly, and that 21 oxen could eat up the whole pasture in 6 weeks, or 18 oxen in 9 weeks; what number of oxen diminished by the removal of 9, at the end of 14 weeks, could eat it up in 18 weeks ?

I. Solution by G. B. M. ZERR. A. M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

In the first case, in one week one ox will eat ($\frac{1}{6}$ of $\frac{8}{21}$ of original grass$+$ $\frac{8}{21}$ of what grows) on one acre$=\frac{4}{63}$ of original grass on one acre$+\frac{8}{21}$ of what grows on one acre.

In the second case, in one week one ox will eat ($\frac{1}{9}$ of $\frac{8}{18}$ of original grass $+\frac{8}{18}$ of what grows) on one acre$=\frac{4}{81}$ of original grass on one acre$+\frac{4}{9}$ of what grows on one acre.

In each case one ox eats the same quantity in one week.

$\therefore\ \frac{4}{9}-\frac{8}{21}=\frac{4}{63}$ of the growth of one acre in one week is $=\frac{4}{63}-\frac{4}{81}=\frac{8}{567}$ of an acre. $\frac{8}{567}\div\frac{4}{63}=\frac{2}{9}$ of an acre, what grows on an acre in one week. $\frac{4}{63}+\frac{8}{21}$ of $\frac{2}{9}=\frac{4}{27}$, the part of the original quantity which one ox eats in one week.

$8\div(6\times\frac{4}{27})=9$ oxen to eat original grass. $21-9=12$ oxen necessary to eat growing grass.

Now change "at the end of 14 weeks" to "at the end of 4 weeks." Otherwise we will have same hungry oxen. 8 acres$=\frac{2}{9}\times18\times8=40$ acres of original grass$=$amount on 8 acres$+$what grows on 8 acres in 18 weeks.

Then $4\times\frac{4}{27}$ oxen$+14\times\frac{4}{27}$ (oxen-9)$=40$.

$\therefore$ 72 oxen$=1080+504=1584$. $\therefore$ oxen$=22$, the number required.

Also solved by *J. R. HITT.*

II. Solution by the PROPOSER.

Let a be the number of pounds of grass on 1 acre, and an that which grows on 1 acre in 1 week. Then 21 oxen eat $(8a+8an\times6)$ pounds in 6 weeks, and 18 oxen eat $(8a+8an\times9)$ pounds in 9 weeks.

$\therefore$ From the first statement, 1 ox eats $\dfrac{4(1+6an)}{63}$ pounds in 1 week.

From the second statement, 1 ox eats $\dfrac{4(1+9an)}{81}$ pounds.

$\therefore\ \dfrac{4(a+9an)}{81}=\dfrac{4(a+6an)}{63}$, whence $n=\frac{2}{9}$.

Let x=the required number of oxen. Then x oxen will eat $\dfrac{56ax}{27}$ pounds in 14 weeks, and $(x-9)$ oxen will eat $\frac{16}{27}a(x-9)$ pounds in 4 weeks.